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Theorem of Total Probability and Baye's Theorem

एक set of events $E_1, E_2, \ldots\ldots, E_n$ is said to represent a partition of the sample space S, अगर वह following conditions को satisfy करेगा :-

(Partition of sample space)

(i) $E_i \cap E_j = \phi$, $i \neq j$; $j, i = 1, 2, \ldots\ldots, n$

(ii) $E_1 \cup E_2 \cup \ldots\ldots \cup E_n = S$

(iii) $P(E_i) > 0$, $\forall i = 1, 2, \ldots\ldots, n$

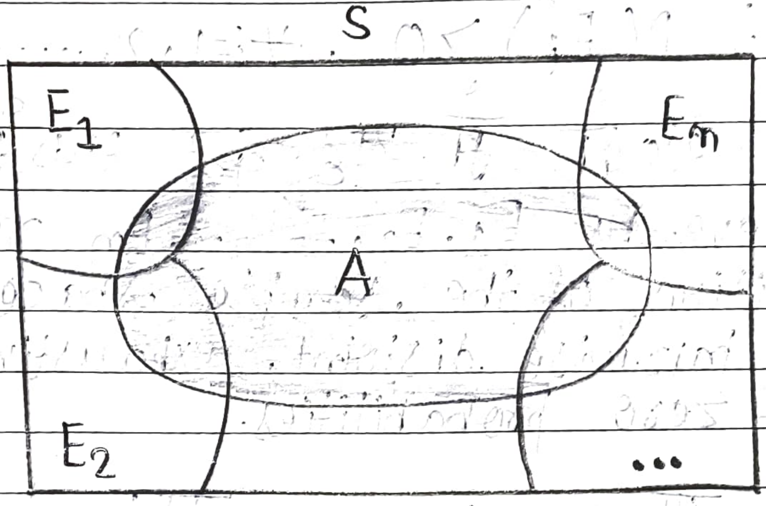
दूसरे शब्दों में यह कह सकते हैं कि :- the events $E_1, E_2, \ldots, E_n$ represent a partition of the sample space S, if they are pairwise disjoint, exhaustive and have non-zero probabilities.

Note: एक sample space का partition unique नहीं होता है, एक sample space का बहुत सारे partitions हो सकता है।

(P.T.O.)

(Theorem of Total Probability)

मान लिया कि $\{E_1, E_2, E_3, \ldots\ldots, E_n\}$ partions हैं sample space S के और मान लिया कि events $E_1, E_2, E_3, \ldots, E_n$ के पास non-zero probability of occurrence है और मान लिया कि A कोई event है जो S से associated है, तब



$$P(A) = P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + \ldots\ldots + P(E_n)\cdot P(A|E_n)$$

$$= \sum_{j=1}^{n} P(E_j)\cdot P(A|E_j)$$

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Baye's Theorem

अगर $E_1, E_2, \ldots, E_n$ हैं 'n' non-empty events जो एक partition बनाता है sample space S का
i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint.

$E_1 \cup E_2 \cup \ldots \cup E_n = S$ and

$P(E_i) > 0, \forall\ i = 1, 2, 3, \ldots, n.$

Also, let A be any event of non-zero probability, तब

$$P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)}, \text{ for any } i = 1, 2, 3, 4, \ldots, n$$

(Or),

मान लो कि $E_1, E_2, E_3, \ldots, E_n$ be 'n' mutually exclusive and exhaustive events जो एक random experiment का हिस्सा है। अगर A एक event है जो E_i's के साथ-साथ occur होगा. तब,

$$P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)}$$

(P.T.O.)

- events $E_1, E_2, \ldots, E_n$ को hypothesis कहेंगे।
- probability $P(E_i)$ को priori probability of the hypothesis E_i कहेंगे।
- Conditional probability $P(E_i|A)$ को posteriori probability of the hypothesis E_i कहेंगे।

Note: Baye's Theorem is also called the formulae for the probability of causes.

Method to solve problem based on Baye's Theorem

baye's theorem पर based problems को solve करने के लिए हम निम्नलिखित steps follow करेंगे :-

I. सभी events को identify करें और उसको naming करें E_1, E_2, E_3 and so on, जो कि एक partition बनाता sample space S को।

II. मान लिया की दूसरा event जिसका non-zero probability है वह 'A' है।

III. सारे events $E_1, E_2, E_3 \ldots\ldots$ का probability निकालेंगे।

IV. A का सारे events $E_1, E_2, E_3, \ldots\ldots$ के साथ conditional probability निकालेंगे।

V. Bayes theorem का use करके हम required probability निकालेंगे i.e. $P(E_i | A)$.

Q. In a bulb factory, three machines A, B, C, manufacture 60%, 25%, and 15% of the total production respectively. Of their respective outputs 1%, 2% and 1% are defective. A bulb is drawn at random from the total product, and it is found to be defective. Find the probability that it was manufactured by machine C.

Ans: Step 1: Let,

E_1: event that bulb is manufactured from ~~factory~~ machine A.

E_2: event that bulb is manufactured from machine B.

E_3: event that bulb is manufactured from machine C.

Step 2: let,

A: event that the bulb drawn ~~drew~~ is defective.

(P.T.O.)

Step 3:

$$P(E_1) = 60\% = \frac{\cancel{60}^{3}}{\cancel{100}_{5}} = \frac{3}{5}$$

$$P(E_2) = 25\% = \frac{\cancel{25}}{\cancel{100}_{4}} = \frac{1}{4}$$

$$P(E_3) = 15\% = \frac{\cancel{15}^{3}}{\cancel{100}_{20}} = \frac{3}{20}$$

Step 4:

$P(A|E_1)$ = probability of defective bulb is made by machine A

= $\frac{\text{favourable outcomes that defective bulb is made by machine A}}{\text{total bulbs produced}}$

$$= \frac{1\%}{100\%} = \frac{1/\cancel{100}}{100/\cancel{100}}$$

$$= \frac{1}{100}.$$

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$P(\sout{E_2 | A})$ = probability of defective bulb is
$A|E_2$ made by machine B.

$= \dfrac{\text{favourable outcomes that defective bulb is made by machine B.}}{\text{total bulbs produced.}}$

$= \dfrac{2\%}{100\%} = \dfrac{2/100}{100/100} = \dfrac{2}{100}$

$P(\sout{E_3 | A})$ = probability of defective bulb is
$A|E_3$ made by machine C.

$= \dfrac{\text{favourable outcomes that defective bulb is made by machine C.}}{\text{total bulbs produced.}}$

$= \dfrac{1\%}{100\%} = \dfrac{1/100}{100/100} = \dfrac{1}{100}$

Step 5:

Now, using the bayes's theorem, we get

$P(E_3 | A)$ = probability that the selected defective bulb is manufactured by C

$$= \frac{P(E_3) \cdot P(A | E_3)}{P(E_1) \cdot P(A | E_1) + P(E_2) \cdot P(A | E_2) + P(E_3) \cdot P(A | E_3)}$$

$$= \frac{\frac{3}{20} \times \frac{1}{100}}{\frac{3}{5} \times \frac{1}{100} + \frac{1}{4} \times \frac{2}{100} + \frac{3}{20} \times \frac{1}{100}}$$

100	200, 2000, 500
5	2, 20, 5
2	2, 4, 1
2	1, 2, 1
	1, 1, 1

$$= \frac{\frac{3}{2000}}{\frac{3}{500} + \frac{2}{400} + \frac{3}{2000}}$$

(400 reduced to 200)

$$= \frac{\frac{3}{2000}}{\frac{12 + 10 + 3}{2000}}$$

$$= \frac{3}{25}$$

Conclusion:

Therefore, the probability of selected defective bulb is from machine C

$$= \frac{3}{25}.$$

Q. A company manufactures scooters at two plants, A and B. plant A produces 80% and plant B produces 20% of the total product. 85% of scooters produced at plant A and 65% of the scooters produced at plant B are of standard quality. A scooter produced by the company is

selected at random, and it is found to be of standard quality. What is the probability that it was manufactured at plant A ?

Ans:

Step 1: let,

E_1: event that a scooter is manufactured at plant A.

E_2: event that a scooter is manufactured at plant B.

Step 2: let,

A: event that scooter is of standard quality.

Step 3:

$$P(E_1) = \frac{80}{100} = \frac{4}{5}$$

$$P(E_2) = \frac{20}{100} = \frac{1}{5}$$

Step 4:

$P(A|E_1)$ = probability that the scooter is of standard quality made by plant A.

= $\frac{\text{favourable outcomes that scooter is of standard quality made by plant A}}{\text{total scooters produced}}$

$$= \frac{\frac{85}{100}}{\frac{100}{100}} = \frac{85}{100}$$

$P(A|E_2)$ = probability that the scooter is of standard quality made by plant B.

$$= \frac{\frac{65}{100}}{\frac{100}{100}} = \frac{65}{100}$$

Step 5:

Now, using baye's theorem, we get

$P(E_1|A)$ = probability that selected standard scooter is from plant A.

$$= \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{\frac{4}{5} \times \frac{85}{100}}{\frac{4}{5} \times \frac{85}{100} + \frac{1}{5} \times \frac{65}{100}}$$

$$= \frac{\frac{17 \times 4}{100}}{\frac{17 \times 4 + 13}{100}} = \frac{17 \times 4}{17 \times 4 + 13}$$

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$$= \frac{68}{68+13} = \frac{68}{81}$$

Q. In a certain college, 4% of boys and 1% of girls are taller than 1.75 meters. Furthermore, 60% of the students are girls. If a student is selected at random and is taller than 1.75 meters, what is the probability that the selected student is a girl?

Ans:

~~Step 1: Let,~~

~~E_1: event that a boy is taller than 1.75 mts.~~
~~E_2: event that a girl is taller than 1.75 mts.~~

~~Step 2:~~

Step 1:
Let,

E_1 : event that a student is a boy.
E_2 : event that a student is a girl.

Step 2:
Let,

A: event that student is taller than 1.75 meters.

(P.T.O.)

Step 3:

$P(E_1) = 40\% = \frac{40}{100}$

$P(E_2) = 60\% = \frac{60}{100}$

Step 4:

$P(A|E_1)$ = Probability that the student is taller than 1·75 meters is a boy

$= \frac{\text{favourable outcomes to the student boy is taller than 1·75 mts.}}{\text{total students.}}$

$= \frac{4\%}{100\%}$

$= \frac{4/100}{100/100} = \frac{4}{100}$

$P(A|E_2)$ = Probability that the student is taller than 1·75 meters is a girl.

$= \frac{\text{favourable outcomes to the student girl is taller than 1·75 mts.}}{\text{total students}}$

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$$= \frac{1\%}{100\%} = \frac{1/100}{100/100} = \frac{1}{100}$$

Step 5:

Now, using the baye's theorem

$P(E_2|A)$ = probability that selected student is a girl and is taller than 1.75 meters.

$$= \frac{P(E_2) \cdot P(A|E_2)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{\left(\frac{60}{100}\right) \times \left(\frac{1}{100}\right)}{\left(\frac{40}{100}\right) \times \left(\frac{4}{100}\right) + \left(\frac{60}{100}\right) \times \left(\frac{1}{100}\right)}$$

$$= \frac{\frac{3}{500}}{\frac{2}{125} + \frac{3}{500}}$$

5	500, 125
5	100, 25
5	20, 5
2	4, 1
2	2, 1
	1, 1

$$= \frac{\frac{3}{500}}{\frac{8+3}{500}} = \frac{3}{11}$$

$125 \times 4 = 500$

Conclusion: Therefore, the probability of selected taller student is a girl is $\frac{3}{11}$.

Q. In a class, 5% of the boys and 10% of the girls have an IQ of more than 150. In this class, 60% of the students are boys. If a student is selected at random and found to have IQ more than 150, find the probability that the student is a boy.

Ans: Step 1:

Let,

E_1: event that a student is a boy.

E_2: event that a student is a girl.

Step 2:

Let,

A: event that student has IQ more than 150.

Step 3:

$P(E_1) =$ ~~5%~~ 60% $P(E_2) = 40\%$

$= \frac{60}{100}$ $= \frac{40}{100}$

Step 4:

$P(A|E_1)$ = Probability that the student has IQ greater than 150 is a boy.

$$= \frac{\text{favourable outcomes to the student boy has more than 150 IQ.}}{\text{total no. of students.}}$$

$$= \frac{5\%}{100\%} = \frac{5/100}{100/100} = \frac{5}{100}$$

$P(A|E_2)$ = Probability that the student has IQ greater than 150 is a girl.

$$= \frac{\text{favourable outcomes to the student girl having IQ more than 150.}}{\text{total no. of students}}$$

$$= \frac{10\%}{100\%} = \frac{10/100}{100/100} = \frac{10}{100} = \frac{1}{100}$$

(P.T.O.)

Step 5:

Now, using the bayer's theorem

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{60}{100}\times\frac{5}{100}}{\frac{60}{100}\times\frac{5}{100} + \frac{40}{100}\times\frac{10}{100}}$$

$$= \frac{\frac{3}{100}}{\frac{3}{100}+\frac{10}{250}}$$

$$= \frac{\frac{3}{100}}{\frac{15+2\times10}{500}}$$

$$= \frac{3\times500}{17\times100}$$

$$= \frac{\frac{3}{100}}{\frac{15+20}{500}}$$

$$= \frac{3 \times \cancel{500}}{\cancel{100} \times \cancel{35}\ 7} = \frac{3}{7}.$$

Conclusion: Therefore, the probability that selected student having IQ greater than 150 is a boy is $\frac{3}{7}$.

Q. Suppose 5% of men and 0.25% of women have grey hair. A grey-haired person is selected at random. What is the probability of this person being male? Assume that there is an equal no. of males and females.

Ans: Step 1:

मान लिया कि

E_1: event that person is a men

E_2: event that person is a woman.

Step 2:

Let,

A: event that the person have grey hair.

Step 3:

$$P(E_1) = \frac{50\%}{100\%} = \frac{50/\cancel{100}}{100/\cancel{100}} = \frac{50}{100}$$

$$P(E_2) = \frac{50\%}{100\%} = \frac{50/\cancel{100}}{100/\cancel{100}} = \frac{50}{100}$$

Step 4:

$P(A|E_1)$ = probability that person have grey hair is a man.

$$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{5\%}{100\%} = \frac{\frac{5}{100}}{100/100} = \frac{5}{100}$$

$P(A|E_2)$ = probability that person have grey hair is a woman.

$$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{\frac{0.25}{100}}{\frac{100}{100}} = \frac{0.25}{100}$$

Step 5:

Now, using the baye's theorem

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)}$$

$$= \frac{\frac{50}{100} \times \frac{5}{100}}{\frac{50}{100} \times \frac{5}{100} + \frac{50}{100} \times \frac{0.25}{100}}$$

$$= \frac{\frac{1}{2} \times \frac{5}{100}}{\left(\frac{1}{2}\right) \times \left(\frac{5}{100}\right) + \left(\frac{1}{2}\right) \times \left(\frac{0.25}{100}\right)}$$

$$= \frac{5}{5.25} = \frac{20}{21}$$

Conclusion: Therefore, the probability of a randomly selected grey person to be male is $\frac{20}{21}$.

Q. Two groups are ~~connecting~~ competing for the positions on the board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4, respectively. Further, if the first group wins, the probability of introducing a new product is 0.7, and when the second groups win, the corresponding probability is 0.3. Find the probability that the new product introduced was by the second group.

(P.T.O.)

Ans: let,

E_1 : be the event that first group will win.
E_2 : be the event that second group will win

A : be the event that new product is introduced.

$P(E_1) = 0.6$ $\qquad$ $P(E_2) = 0.4$

$P(A|E_1)$ = probability that new product is introduced ~~by~~ when first group will wins.

$= \frac{0.7}{1}$ ~~P(A) · P(~~

$P(A|E_2)$ = probability that new product is introduced when second group will wins.

$= \frac{0.3}{1}$

by using baye's theorem, we get

$$P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{(0.4 \times 0.3)}{(0.6 \times 0.7) + (0.4 \times 0.3)}$$

$$= \frac{0.12}{0.42 + 0.12} = \frac{0.12}{0.54} = \frac{2}{9}$$

Conclusion: Therefore, the probability of the second group introduced a new product

$$= \frac{2}{9}.$$

Q. A bag A contains 1 white and 6 red balls. Another bag B contains 4 white and 3 red balls. One of the bags is selected at random, and a ball is drawn from it, which is found to be white. Find the probability that the ball drawn is from bag A.

Ans: Let,

E_1 = event that ball drawn is red [redacted]

E_2 = event that ball drawn is white [redacted]

E_3 = event that [redacted] bag B selected

E_4 = event that [redacted] bag A selected

~~$P(E_1) =$~~

$P(E_3) = P(E_4) = \frac{1}{2}$ { assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$

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$P(E_2|E_3)$

~~$P(E_3|E_2)$~~ = probability that bag B is selected and ~~red~~ white ball is drawn

$= \frac{4}{7}$.

$P(E_2|E_4)$

~~$P(E_4|E_2)$~~ = probability that bag A is selected and white ball is drawn

$= \frac{1}{7}$.

by using baye's theorem, we get

$P(E_4|E_2)$ = probability that selected white ball is from bag A.

$$= \frac{P(E_4)\cdot P(E_2|E_4)}{P(E_3)\cdot P(E_2|E_3) + P(E_4)\cdot P(E_2|E_4)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{1}{7}} = \frac{1}{5}$$

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Conclusion: Therefore, the probability of selected white ball is from bag A.

$$= \frac{1}{5}$$

Q. There are two bags I and II. The bag I contains 3 white and 4 black balls, and bag II contains 5 white and 6 black balls. One ball is drawn at random from one of the bags and is found to be white. Find the probability that it was drawn from the bag I.

Ans: मान लिया कि :-

W : white ball is drawn.
B : Black ball is drawn.
X : 1st bag is selected.
Y : 2nd bag is selected.

Assuming, selecting bags is equal probability i.e. $\frac{1}{2}$

$$P(X) = P(Y) = \frac{1}{2}.$$

$P(W|X)$ = probability that white ball is drawn when ~~2nd~~ 1st bag is selected

$$= \frac{3}{7}.$$

(P.T.O.)

$P(W|Y)$ = probability that white ball is drawn when 2nd bag is selected.

$$= \frac{5}{11}.$$

by using baye's theorem, we get

$P(X|W)$: probability of selected white ball is from the 1st bag.

$$P(X|W) = \frac{P(X) \cdot P(W|X)}{P(X) \cdot P(W|X) + P(Y) \cdot P(W|Y)}$$

$$= \frac{\left(\frac{1}{2}\right) \times \left(\frac{3}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{7}\right) + \left(\frac{1}{2}\right) \times \left(\frac{5}{11}\right)}$$

$$= \frac{\frac{3}{7}}{\frac{3}{7} + \frac{5}{11}}$$

$$= \frac{33}{68}.$$

Conclusion: Therefore, the probability of selected white ball is from the 1st bag is $\frac{33}{68}$.

Q. A box contains 2 gold and 3 silver coins. Another box contains 3 gold and 3 silver coins. A box is chosen at random, and a coin is drawn from it. If the selected coin is a gold coin, find the probability that it was drawn from the second box.

Ans: माना किया कि :—

G : Gold coin is ~~selected~~ drawn.
S : Silver coin is ~~selected~~ drawn.
A : 1st box is choosen.
B : 2nd box is choosen.

Assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$.

So, $P(A) = P(B) = \frac{1}{2}$.

$P(G|A)$ = Probability that a gold coin is drawn when 1st box is chosen.

$= \frac{2}{5}$

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$P(G|B)$ = probability that a gold coin is drawn when 2nd box is chosen.

$$= \frac{3}{6} = \frac{1}{2}.$$

by using baye's theorem, we get

$P(B|G)$ = Probability that gold coin is selected from the 2nd box.

$$= \frac{P(B) \cdot P(G|B)}{P(A) \cdot P(G|A) + P(B) \cdot P(G|B)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{2}{5} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}}$$

$$= \frac{5}{9}$$

Conclusion: Therefore, the probability of selected gold coin is from the 2nd box

$$= \frac{5}{9}.$$

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Q. Three urns A, B and C contains 6 red and 4 white; 2 red and 6 white; and 1 red and 5 white balls respectively. An urn is chosen at random, and a ball is drawn. If the ball drawn is found to be red, find the probability that the balls was drawn from the first urn A.

Ans: मान लिया कि :-

A: event that urn A is choosen.
B: event that urn B is choosen.
R: event that red ball is drawn.
W: event that white ball is drawn.
C: event that urn C (urn) is choosen.

let's assume that, choosing is of equal probabilitity i.e. $\frac{1}{2}$.

$P(A) = P(B) = \frac{1}{3} = P(C)$

$P(R|A)$ = Probability that the red ball is drawn when urn A is choosen.

$= \frac{6}{10}$

$P(R|B)$ = Probability that the red ball is drawn when urn B is choosen.

$= \frac{2}{8}$.

(P.T.O.)

$P(R|C)$ = Probability that the red ball is choosen when urn C is choosen.

$$= \frac{1}{6}$$

Now, by using baye's theorem, we get

$P(A|R)$ = Probability that red ball is selected from urn A.

$$= \frac{P(A) \cdot P(R|A)}{P(A) \cdot P(R|A) + P(B) \cdot P(R|B) + P(C) \cdot P(R|C)}$$

$$= \frac{\left(\frac{1}{2} \times \frac{6}{10}\right)}{\left(\frac{1}{2} \times \frac{6}{10}\right) + \left(\frac{1}{2} \times \frac{2}{8}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right)}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{1}{8} + \frac{1}{12}}$$

10, 8, 12

2 | 5, 4, 6

24 × 5 = 120

$$= \frac{\frac{3}{10}}{\frac{3 \times 12 +}{120}}$$

$$= \frac{\left(\frac{1}{3} \times \frac{6}{10}\right)}{\left(\frac{1}{3}\right) \times \left(\frac{6}{10}\right) + \left(\frac{1}{3}\right) \times \left(\frac{2}{8}\right) + \left(\frac{1}{3}\right) \times \left(\frac{1}{6}\right)}$$

$$= \frac{\frac{3}{5}}{\frac{3}{5} + \frac{1}{4} + \frac{1}{6}} = \frac{36}{61}$$

Conclusion: Therefore, the probability of selected red ball is from bag A is

$$= \frac{36}{61}.$$

Q. Three urns contains 2 white and 3 black balls; 3 white and 2 black balls, and 4 white and 1 black ball respectively. One ball is drawn from an urn chosen at random, and it was found to be white. Find the probability that it was drawn from the first urn.

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Ans: मान लिया कि :-

A : First urn is selected.
B : second urn is selected.
C : Third urn is selected.
BB : black ball is drawn.
WB : white ball is drawn.

Assuming, selecting bags is of equal probability i.e. $\frac{1}{3}$

So,

$$P(A) = P(B) = P(C) = \frac{1}{3}.$$

~~Now, using bayes theorem, we get~~

$P(WB|A)$ = Probability that white ball is drawn when first urn is selected.

$$= \frac{2}{5}.$$

$P(WB|B)$ = Probability that white ball is drawn when second urn is selected.

$$= \frac{3}{5}.$$

$P(WB|C)$ = Probability that white ball is drawn when third urn is selected.

$$= \frac{4}{5}$$

Now, by using baye's theorem; we get

$$P(A|WB) = \frac{P(A) \cdot P(WB|A)}{P(A) \cdot P(WB|A) + P(B) \cdot P(WB|B) + P(C) \cdot P(WB|C)}$$

$$= \frac{\left(\frac{1}{3}\right) \times \left(\frac{2}{5}\right)}{\left(\frac{1}{3}\right) \cdot \left(\frac{2}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{3}\right) \cdot \left(\frac{4}{5}\right)}$$

$$= \frac{2}{9}$$

Conclusion: Therefore, the probability of selected white ball is from bag A.

$$= \frac{2}{9}$$

Q. There are three boxes, the first one containing 1 white, 2 red and 3 black balls; the second one containing 2 white, 3 red and 1 black ball and the third one containing 3 white, 1 red & 2 black balls. A box is chosen at random, and from it, two balls are drawn at random. One ball is red and the other, white. What is the probability that they

come from the second box ?

Ans: मान लिया कि :-

A : bag A is selected.
B : bag B is selected.
C : bag C is selected.
BB : black ball is drawn.
W[redacted] : white ball is drawn.
R[redacted] : red ball is drawn.

Assuming, selecting the bag is of equal probability i.e. $\frac{1}{3}$

So,

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$P(WR|A)$ = probability that white and red ball is drawn when bag A is selected.

$$= \frac{3}{56}$$

$P(WR|B)$ = probability that white and red ball is drawn when bag B is selected

$$= \frac{5}{6}$$

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$P(WR|C)$ = probability that white and red ball is drawn when bag C is selected

$$= \frac{4}{6} = \frac{2}{3}$$

Now, by using Baye's theorem, we get

$$P(B|WR) = \frac{P(B) \times P(WR|B)}{P(A) \cdot P(WR|A) + P(B) \cdot P(WR|B) + P(C) \cdot P(WR|C)}$$

$$= \frac{\frac{1}{3} \times \frac{5}{6}}{\frac{1}{3} \times \frac{3}{6} + \frac{1}{3} \times \frac{5}{6} + \frac{1}{3} \times \frac{2}{3}}$$

3	18, 9, 18
2	6, 3, 6
3	3, 3, 2
2	1, 1, 2
	1, 1, 1

$$= \frac{\frac{5}{18}}{\frac{3}{18} + \frac{5}{18} + \frac{2}{9}}$$

$$= \frac{\frac{5}{18}}{\frac{6+10+8}{36}}$$

$$= \frac{5 \times 36^{2}}{_{12}24 \times 18} = \frac{5}{12}.$$

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Conclusion: Therefore,

the probability of selected white and red ball from bag B

$$= \frac{5}{12}$$

Q. Urn A contains 7 white and 3 black balls; urn B contains 4 white and 6 black balls; urn C contains 2 white and 8 black balls. One of these urns is chosen at random with probabilities 0.2, 0.6 and 0.2 respectively. From the chosen urn, two balls are drawn at random without replacement. Both the balls happen to be white. Find the probability that the balls are drawn from urn C.

Ans: मान लिया कि :-

A: bag A is choosen
B: bag B is choosen
C: bag C is choosen
Z: black ball is drawn
X: white ball is drawn
Y: red ball is drawn

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given,

$P(A) = 0.2$; $P(B) = 0.6$; $P(C) = 0.2$

$P(X|A)$ = Probability that white ball is drawn when bag A is choosen.

$$= \frac{^{7}C_2}{^{10}C_2}$$

$^{7}C_2$ (No. of ways of choosing/drawing 2 balls out of 7 white balls)

$^{10}C_2$ (No. of ways of drawing 2 ball out of total 10 balls in urn A)

$P(X|B)$ = Probability that white ball is drawn when bag B is choosen.

$$= \frac{^{4}C_2}{^{10}C_2}$$

$^{4}C_2$ (No. of ways of drawing 2 balls out of 4 white balls)

$^{10}C_2$ (No. of ways of drawing 2 balls out of 10 balls in total from urn B)

$P(X|C)$ = Probability that white ball is drawn when bag C is choosen.

$$= \frac{^{2}C_2}{^{10}C_2}$$

$^{2}C_2$ (No. of ways of drawing 2 balls out of 2 white balls available ~~in total~~ in bag C)

$^{10}C_2$ (No. of ways of drawing 2 balls out of 2 ~~white~~ balls from 10 available ball in urn C)

Now, using baye's theorem, we get

P(C | X) = Probability that white ball is drawn when bag C is choosen.

$$= \frac{P(C) \cdot P(X|C)}{P(A) \cdot P(X|A) + P(B) \cdot P(X|B) + P(C) \cdot P(X|C)}$$

$$= \frac{0.2 \times \frac{1}{45}}{0.2 \times \frac{21}{45} + 0.6 \times \frac{6}{45} + 0.2 \times \frac{1}{45}}$$

$$= \frac{1}{40}$$

Conclusion:

Therefore, the probability of both selected white balls are from urn C is $\frac{1}{40}$.

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Q. There are 3 bags, each containing 5 white and 3 black balls. Also, there are 2 bags, each containing 2 white and 4 black balls. A white ball is drawn at random. Find the probability that this ball is from a bag of the first group.

Ans: मान लिया कि :-

A: set of first 3 bags.
B: a set of next 2 bags.
W: white ball is drawn.
Z: black ball is drawn.

अब हम problem को change करके दो bag का बना देते हैं i.e. bag A में 15 white और 9 black balls हैं (5 white & 3 black in each bag) and bag B में 4 white and 8 black balls हैं (2 white & 4 black in each bag)

अब,

Probability of selecting bag A $= \frac{3}{5}$ (3 bags are in A)

Probability of selecting bag B $= \frac{2}{5}$ (2 bags are in B)

अब हमको निकालना है

P(W|A) = Probability that white ball is drawn when the first set of 3 bags are taken.

$$= \frac{15}{24}$$

$P(W|B)$ = Probability that the white ball is drawn when the next set of 2 bags are taken

$$= \frac{4}{12}$$

Now, using the baye's theorem, we get

$P(A|W)$ = Probability that the ball is drawn from first set of 3 bag if the ball is white.

$$= \frac{P(A) \cdot P(A|W)}{P(A) \cdot P(A|W) + P(B) \cdot P(B|W)}$$

$$= \frac{\left(\frac{3}{5}\right) \times \left(\frac{15}{24}\right)}{\left(\frac{3}{5}\right) \times \left(\frac{15}{24}\right) + \left(\frac{2}{5}\right) \times \left(\frac{4}{12}\right)}$$

$$= \frac{45}{61}$$

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Conclusion:

Therefore, the probability of selected white ball is from the first group is

$$= \frac{45}{61}$$

Q. There are four boxes A, B, C and D, containing marbles. A contains 1 red, 6 white and 3 black marbles; B contains 6 red, 2 white and 2 black marbles; C contains 8 red, 1 white and 1 black marbles; and D contains 6 white and 4 black marbles. One of the boxes is selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from the box A?

Ans: मान लिया कि :—

A : event that bag A is choosen.
B : event that bag B is choosen.
C : event that bag C is choosen.
D : event that bag D is choosen.
X : event that ball drawn is black.
Y : event that ball drawn is white
Z : event that ball drawn is red.

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Assuming all boxes have equal probability for picking i.e. $\frac{1}{4}$.

So,

$$P(A) = P(B) = P(C) = P(D) = \frac{1}{4}.$$

$P(Z|A)$ = Probability that ball drawn is red if bag A is choosen

$$= \frac{1}{10}$$

$P(Z|B)$ = Probability that ball drawn is red if bag B is choosen

$$= \frac{6}{10}$$

$P(Z|C)$ = Probability that ball drawn is red if bag C is choosen

$$= \frac{8}{10}$$

$P(Z|D)$ = Probability that ball drawn is red if bag D is choosen

$$= \frac{0}{10}.$$

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by, using baye's theorem :-

$P(A|Z)$ = Probability that ball drawn is red from the box A

$$= \frac{P(A)\cdot P(Z|A)}{P(A)\cdot P(Z|A) + P(B)\cdot P(Z|B) + P(C)\cdot P(Z|C) + P(D)\cdot P(Z|D)}$$

$$= \frac{\left(\frac{1}{4}\right)\times\left(\frac{1}{10}\right)}{\left(\frac{1}{4}\right)\cdot\left(\frac{1}{10}\right) + \left(\frac{1}{4}\right)\times\left(\frac{6}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{8}{10}\right) + \left(\frac{1}{4}\right)\times\left(\frac{0}{10}\right)}$$

$$= \frac{1}{15}$$

Conclusion: Therefore, the probability of selected red ball is from box A is $\frac{1}{15}$

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Q. A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of the cars, and plant Y manufactures 30%. At plant X, 80% of the cars are rated of standard quality & at plant Y 90% rated standard quality. A car is picked up at random and is found to be of standard quality. Find the probability that it has come from plant X.

Ans: माना कि :-

X: Car produced from plant X.
Y: Car produced from plant Y.
S: Car rated as standard quality.

$$P(X) = \frac{70\%}{100\%} = \frac{70/100}{100/100} = \frac{70}{100}$$

$$P(Y) = \frac{30\%}{100\%} = \frac{30/100}{100/100} = \frac{30}{100}$$

P(S | X) = Probability that the car rated as standard quality is produced from plant X.

$$= \frac{80\%}{100\%} = \frac{80/100}{100/100} = \frac{80}{100}$$

$P(S|Y)$ = Probability that car rated as standard quality produced in plant Y.

$$= \frac{90\%}{100\%} = \frac{90/100}{100/100} = \frac{90}{100}$$

Now, by using bayes's theorem, we get

$P(X|S)$ = Probability that a car produced from plant X is of standard quality.

$$= \frac{P(X) \cdot P(S|X)}{P(X) \cdot P(S|X) + P(Y) \cdot P(S|Y)}$$

$$= \frac{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right)}{\left(\frac{70}{100}\right) \times \left(\frac{80}{100}\right) + \left(\frac{30}{100}\right) \times \left(\frac{90}{100}\right)}$$

$$= \frac{56}{83}.$$

Conclusion: Therefore, the probability of selected standard quality car is from plant X is

$$\frac{56}{83}.$$

Q An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01, and that of motorcycles is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle.

Ans: <u>मान लिया कि :-</u>

M : Motorcycle
S : Scooter
A : Accidented vehicle.

$P(M) = \frac{3000}{5000}$; $P(S) = \frac{2000}{5000}$

$P(A|M)$ = probability that accidented vehicle was a motorcycle

$= 0.02$

$P(A|S)$ = probability that accidented vehicle was a scooter

$= 0.01$.

~~Conclusion: Therefore, the probability of selected standard quality car is from plant X.~~

~~$= \frac{56}{83}$~~

by bayes theorem, we get :-

P(↑

$P(S|Y)$ = Probability that the car rated as standard quality is produced from plant Y.

$$= \frac{90\%}{100\%} = \frac{90/100}{100/100} = \frac{90}{100}.$$

by using bayes theorem, we get

$P(X|S)$ = Probability that car produced from plant X is of standard quality.

$$= \frac{P(X) \cdot P(S|X)}{P(X) \cdot P(S|X) + P(Y) \cdot P(S|Y)}$$

$$= \frac{\left(\frac{70}{100}\right) \times \left(\frac{80}{100}\right)}{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right) + \left(\frac{30}{100}\right) \times \left(\frac{90}{100}\right)} = \frac{56}{83}$$

Conclusion: Therefore, the probability that selected standard quality car is from plant X

$$= \frac{56}{83}.$$

Q. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01, and that of motorcycles is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle.

Ans: माना लिया कि :—

M : Motorcycle.
S : Scooter
A : Accidented vehicle.

$P(M) = \frac{3000}{5000}$ $\quad P(S) = \frac{2000}{5000}$

$P(A|M)$ = Probability that accidented vehicle was a motorcycle.

$= 0.02$

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P(A|S) = Probability that accidented vehicle was a scooter.

= 0.01.

by using bayes's theorem, we get

P(M|A) = Probability that accidented vehicle was a motorcycle.

$$= \frac{P(M) \cdot P(A|M)}{P(M) \cdot P(A|M) + P(S) \cdot P(A|S)}$$

$$= \frac{\left(\frac{3000}{5000}\right) \times \frac{0.02}{100}}{\left(\frac{3000}{5000}\right) \times \frac{0.02}{100} + \left(\frac{2000}{5000}\right) \times \frac{0.01}{100}}$$

$$= \frac{6}{8} = \frac{3}{4}$$

Conclusion: Therefore,

the probability of accidented vehicle was a motorcycle

$$= \frac{3}{4}.$$

Q. In a bulb factory, machines A, B and C manufactures 60%, 30% and 10% bulbs respectively. Out of these bulbs 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by machine A.

Ans: माना लिया कि :—

A : Manufactured from machine A.
B : Manufactured from machine B.
C : Manufactured from machine C.
D : Defective bulb.

$$P(A) = \frac{60\%}{100\%} = \frac{60/\cancel{100}}{100/\cancel{100}} = \frac{60}{100}$$

$$P(B) = \frac{30\%}{100\%} = \frac{30/\cancel{100}}{100/\cancel{100}} = \frac{30}{100}$$

$$P(C) = \frac{10\%}{100\%} = \frac{10/\cancel{100}}{100/\cancel{100}} = \frac{10}{100}$$

P(D|A) = Probability that a defective bulb is manufactured from Machine A.

$$= 1\% = \frac{1}{100}$$

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$P(D|B)$ = Probability that a defective bulb is manufactured from Machine B.

$$= 2\% = \frac{2}{100}$$

$P(D|C)$ = Probability that a defective bulb is manufactured from Machine C.

$$= 3\% = \frac{3}{100}.$$

therefore; by baye's theorem, we get

$P(A|D)$ = Probability that a defective bulb is manufactured from machine A.

$$= \frac{P(A) \cdot P(D|A)}{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)}$$

$$= \frac{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{2}{100}\right) + \left(\frac{10}{100}\right)\left(\frac{3}{100}\right)}$$

$$= \frac{6}{15} = \frac{2}{5}$$